[MATHS PROBLEM]

VENN DIAGRAMS FOR PROBABILITY

Calculating probabilities using Venn diagrams can often lead to confusion, says **Colin Foster**

In this lesson, students learn how probabilities can be calculated by using the correct regions of a Venn diagram.

THE DIFFICULTY

I want you to estimate the probability that if I select a person from this school at random, they will be a football player.

Students might be unsure what 'at random' means. We just mean that everyone has the same chance of being selected. They might also query who counts as a 'football player' – someone who belongs to a team, or just someone who plays occasionally? These are good questions to ask, so agree on some definition.

Students might answer in words (e.g. 'very likely') or give fractions or percentages. Agree on some sensible value, such as 20%.

*Now estimate the probability that if I select someone randomly from this school they will be a **hockey** player.*

*Now estimate the probability that if I select someone randomly from this school they will be **either** a football player **or** a hockey player.*

See whether students think they should add the previous two probabilities or not.

THE SOLUTION

Rather than our whole school, let's imagine just a small class of 10 students. Here they are on a Venn diagram
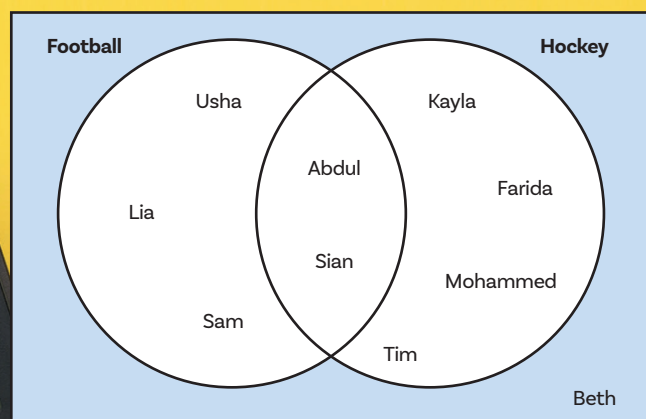
What does this diagram tell you about Abdul? That he does football and hockey.
What does this diagram tell you about Beth? That she does neither football nor hockey.
What does this tell you about Usha? That she does football – **and not hockey**.
Which is more popular – football or hockey? Hockey (6 people versus 5 for football).

If I select someone from the class at random, what's the probability they will do football?
Students might think that the answer is $\frac{3}{5}$ or $\frac{3}{10}$, but it's $\frac{5}{10} = \frac{1}{2} = 50\%$. To answer this they need to **disregard** whether someone plays hockey or not, which means completely ignoring the boundary created by the right-hand circle.

If I select someone from the class at random, what's the probability they will do hockey?
The answer is $\frac{6}{10} = \frac{3}{5} = 60\%$.

*If I select someone from the class at random, what's the probability they will do **either** football **or** hockey?*
The answer can't be $50\% + 60\%$, because that would be more than 100%, which is impossible! The answer is that 9 people do either sport, out of 10 people altogether, so the answer is $\frac{9}{10} = 90\%$. Another way to see this is that it's everyone **except** Beth, who is $\frac{1}{10}$ of all of the people, so $\frac{9}{10}$ of the people do at least one sport.

*If I select someone from the class at random, what's the probability they will do **both** football **and** hockey?*
This is just Abdul and Sian, and the probability of selecting either of them is $\frac{2}{10} = 20\%$.

Checking for understanding
Make up a Venn diagram with two partially overlapping categories and 12 people altogether. Write the people's names in whichever regions of the diagram you choose. Make up 6 probability questions about your Venn diagram, similar to the ones you've seen in this lesson. Write out the answers clearly. See if your partner can answer your questions.

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