[MATHS PROBLEM]

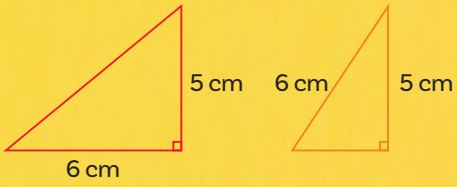
PYTHAGORAS'S THEOREM

When using Pythagoras's Theorem, students often muddle up calculating a hypotenuse and calculating a leg.

In this lesson, students distinguish a hypotenuse from a leg by repeatedly switching between them.

THE DIFFICULTY

What's the same and what's different about these two triangles?



Students might pick up on the different colours, that they are both right-angled, that the two given lengths are the same but differently placed (6 cm is a leg in the red triangle and a hypotenuse in the orange triangle), and that the areas and perimeters are different.

Which triangle has the larger perimeter?
It isn't necessary to do any calculations to reason that the red triangle is larger than the orange one. The largest side in a right-angled triangle is always the hypotenuse. In the orange triangle, this is 6 cm, so the perimeter is 6 + 5 + something ***smaller*** than 6 cm. In the red triangle, the perimeter is 6 + 5 + something ***larger*** than 6 cm. So, the red triangle has the larger perimeter.

Work out the perimeters of these triangles.
The missing sides are red: $\sqrt{6^2 + 5^2} = \sqrt{61} = 7.810$ cm (3 dp) and orange: $\sqrt{6^2 - 5^2} = \sqrt{11} = 3.317$ cm (3 dp), so the perimeters are red: 18.810 cm (3 dp) and orange: 14.317 cm (3 dp).

Watch out for students being unsure about whether they need to square and ***add*** or square and ***subtract***.

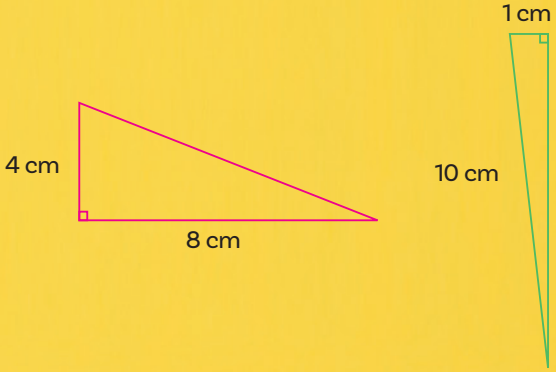
THE SOLUTION

With the red and orange triangles, we could see without calculating which one must have the larger perimeter. But that isn't always the case. Which of these triangles has the larger perimeter?

Students might compare the 5 cm with the 2 cm and conclude that the orange one has the greater perimeter, but this is wrong because the 6 cm is the hypotenuse in the orange triangle but a leg in the blue triangle.

This time we need to calculate. We already know the perimeter of the orange triangle, but the blue one is new, and its perimeter comes to $\sqrt{6^2 + 2^2} + 6 + 2 = 14.325$, which is just slightly ***longer*** than the orange perimeter – too small to detect by eye.

Now try these two:

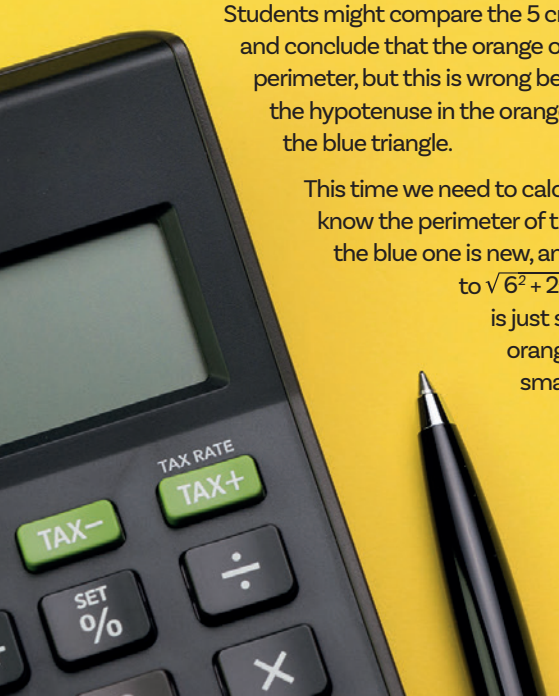


These are very close: pink perimeter = 20.944 cm and green perimeter = 20.950 cm (3dp). Students might struggle to see that the 10 cm side in the green one is a hypotenuse and not a leg. It must be the hypotenuse, because it is ***opposite the right angle.***

Checking for understanding

To assess students' understanding of Pythagoras calculations, ask them to order the triangles on the task sheet (available at bit.ly/ts114-mp1) by perimeter, starting with the smallest.

The answer is C < A < B < H < G < F < E < D.



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