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Introduction

This book arose from comments made by school pupils that lessons in other subjects often feel more varied than those in mathematics. In English lessons, for example, activities might regularly include:

- creative writing, either individually or collaboratively
- improvisation or performance in the drama studio
- whole-class or individual reading of plays, novels, poems and short stories
- watching films, documentaries and televised plays and novels
- formal and informal debates
- whole-class or group discussions
- analysis of somebody else's writing
- exercises on spelling, grammar or punctuation

For a lot of students it seems that too many mathematics lessons feel like the last item on the list. To them the lessons appear dry, predictable, routine and uninspiring. Yet there is no need for mathematics to be experienced in such a dull manner, as there are countless more imaginative ways of structuring mathematics lessons.

Variety in Mathematics Lessons collects together some more interesting lesson models. Although the book includes examples of lessons, it is not merely a set of lesson plans. In a way, it focuses on the stage *before* a lesson plan is formed – more like *a plan for a lesson plan*.

Nothing here should be taken as prescriptive. Professionals in their particular classrooms are the only people who are in the position to know their learners. Only they can take account of the many factors concerning the pupils and their teacher at a given moment. This book offers a collection of rough 'templates' for different genres of mathematics lesson. If things are feeling a bit samey, it might be time for a different 'sort' of lesson, regardless of the subject matter you wish to work on. This book focuses on particular lesson models that have been enjoyable to teach, but it is not by any means meant to be a comprehensive catalogue of lesson types.

There are often strong pressures on teachers towards consistency, confining lessons to predetermined moulds. This is felt to provide a safe and comfortable structure for both learners and teachers, yet can lead to an uninspiring uniformity. It is far better to aim for surprise, flexibility, diversity and variety in mathematics lessons.

The main idea for each lesson is introduced with the light bulb icon.

Dealing with the aftermath – the follow-up to the main thrust of the lesson – is identified by the footprints.

Further avenues that can be explored and extensions of the main idea are in tinted boxes with the arrow icon.

2 It must be one or the other

The sense that two mutually contradictory things cannot both be true simultaneously goes very deep in human beings. So a situation in which there is the potential to see things in two clashing ways can be useful in developing understanding, especially if some learners take one view and others the opposite. The necessity to choose one way or the other can push thinking forward. A tactile or sensory 'feel' to the context can also be helpful, rooting the discussion in 'reality'.

Example 2.1 **Ratio**

A good setting for talking about ratio is one in which the thing that is the same when the 'ratio' is the same is easily *felt*. Taste is a natural possibility, especially if both of the components of the mixture have some easily describable quality (i.e., neither is water).

Suppose you mix 10 litres of orange juice and 4 litres of lemonade. That makes 14 litres of fizzy orange. Another time there are more people, so you mix 11 litres of orange juice with 5 litres of lemonade, to make 16 litres altogether.

Will the drinks taste the same or different? Why?

Taste is sufficiently 'grounded' for learners to feel that it must be either the same or different – it is unusual for a young person not to have an opinion on this. If different, will it taste more 'orangey' or more 'fizzy'? Discussion normally makes reference to other possible mixtures, so a table such as the one below can be an aid to discussion, filling in the quantities as various possibilities are considered.

	A	B				
orange	10	11				
lemonade	4	5				

Thrashing out different ideas can cement why the concept of ratio is a helpful way of looking at a situation such as this. Answers involving the word 'ratio' tend not to help!

Follow-up questions might include:

- *Which ones will taste the same as this one? Why?*
- *Which ones will taste more 'orangey' and which ones will taste more 'lemonadey'?*

Paint is another obvious context (and red and white may be easier colours to talk about than, say, blue and white, because of the availability of the words 'pink' and 'pinkiness'). However, the more vivid sensory aspect of taste seems to make for a more powerful generator of opinions than the perhaps vaguer visual impression of colour.

Example 2.2 **Percentages**

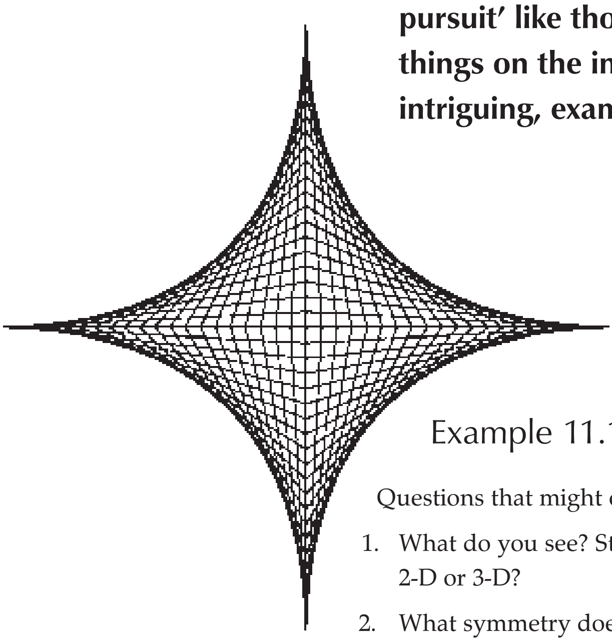
There are plenty of easy ways to start an argument here. For example:

Is a 10% increase followed by a 20% decrease the same as a 20% decrease followed by a 10% increase. If not, is it more or less?

How does a 10% increase followed by a 10% increase compare with a 20% increase?

11 Stimulus – response

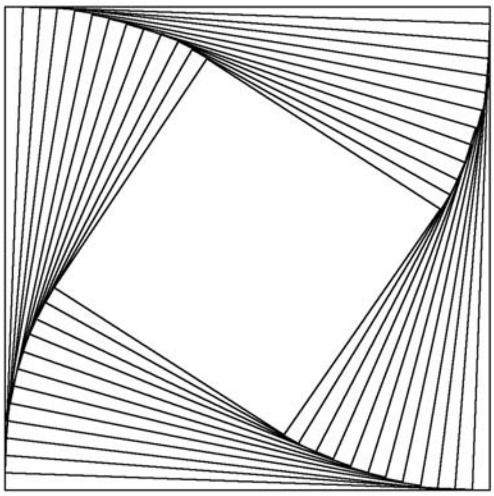
This sort of lesson crystallises out of some initial object of inspiration. It could be a picture, such as a curve-stitching pattern or a 'curve of pursuit' like those shown. There are many beautiful images of such things on the internet as well as some more mundane, though still intriguing, examples.



Example 11.1 **Curve-stitching and curves of pursuit**

Questions that might occur include:

1. What do you see? Straight lines or curves? 2-D or 3-D?
2. What symmetry does the image have?
3. How do you think it was made?
4. Could you make one? What equipment would you need to do it by hand? What software would help? Have a go.
5. What mathematical ideas are buried in it? Do the curves consist of circular arcs?
6. What can you change to make new images?
7. What other questions can you ask?



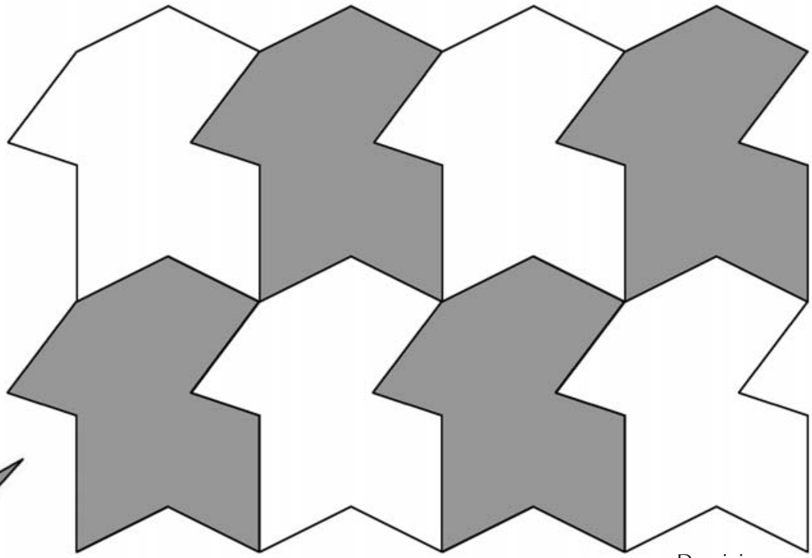
Example 11.2 **Anamorphic art**

Often the first impulse on seeing designs such as these is the simple desire to reproduce the given pattern. This is not necessarily a waste of time. While this is going on, in addition to practising accurate drawing using geometrical instruments, discussion can take place regarding the mathematical properties of the construction.

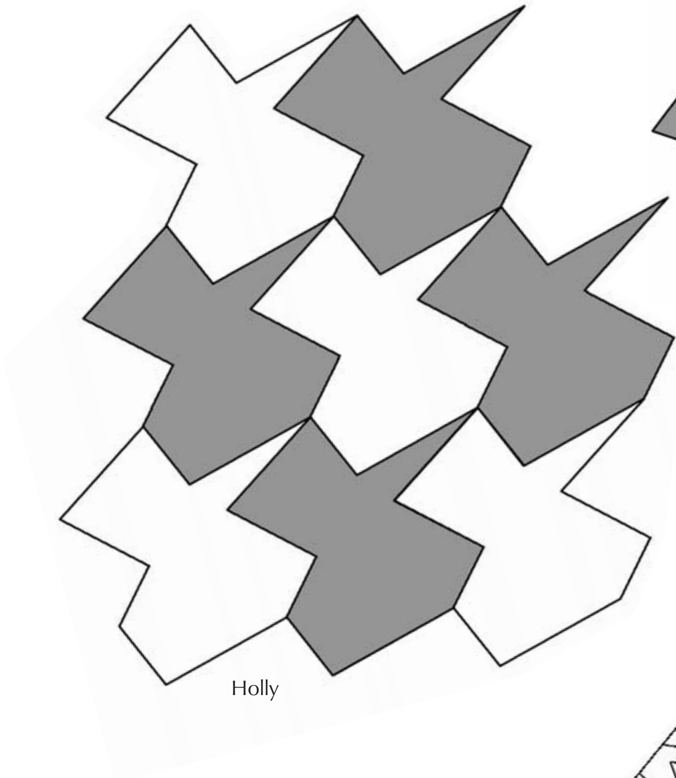
Go to http://users.skynet.be/J.Beever/pave.htm to see more photographs of Julian Beever's amazing anamorphic pavement drawings. What mathematics do they contain? What questions can you ask?

Example 11.3 **Tessellations**

There is a great deal of beautiful and inspiring mathematical artwork by Maurits Cornelis Escher (1898-1972) at **www.mcescher.com**. Books and posters for the classroom are widely available and can provoke a great deal of thought and reflection.



Dominic

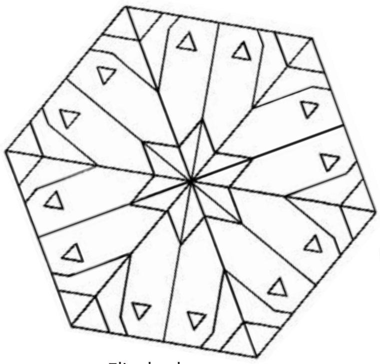


Holly

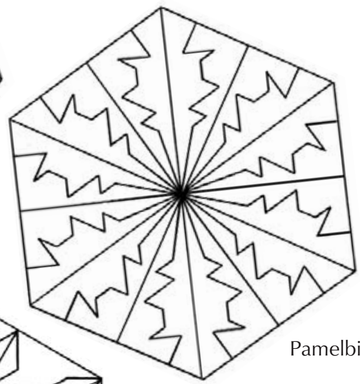
- What is the same about both of these tessellating patterns? What is different?
- What is the same about the repeating units in both patterns? What is different?
- How do you think these patterns were made?
- Could you make some designs like these? How?
- Could you make some tessellating designs where some, or all, of the lines are curves? How?

Example 11.4 **Snowflakes**

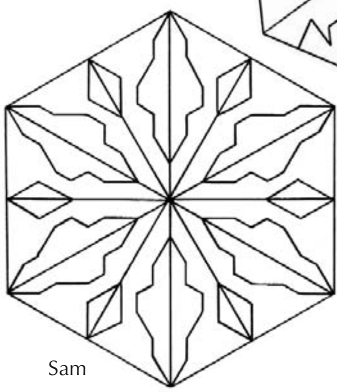
- What occurs to you when you look at these drawings?
- How do you think they were made? Why?
- What could you change without destroying the idea? What would stay the same?
- Could you reproduce them? Could you improve them? How?



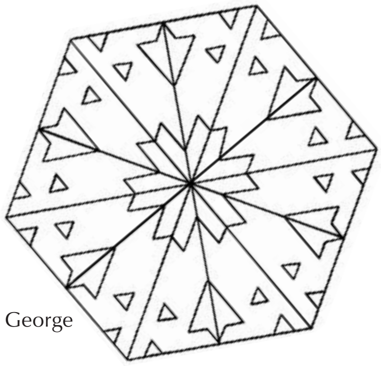
Elizabeth



Pamelbir

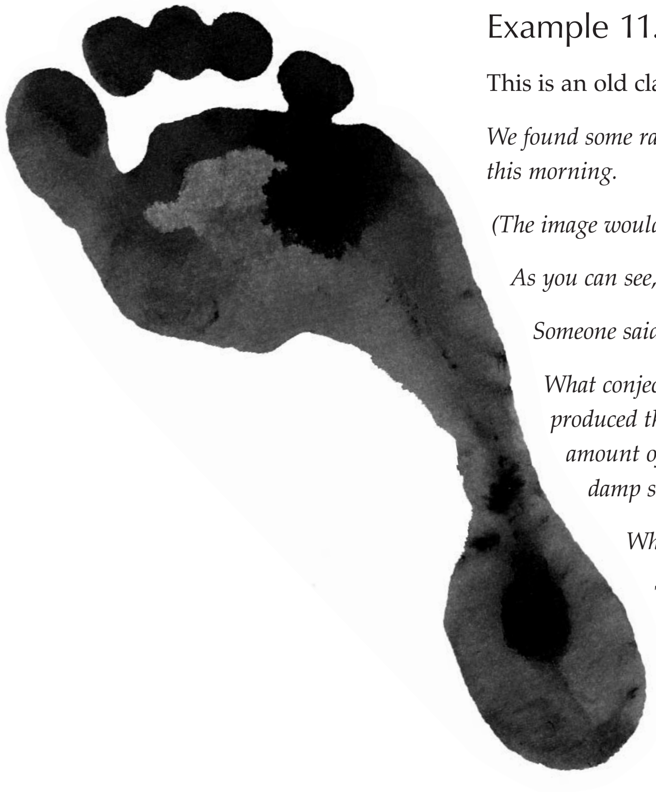


Sam



George

Designs such as these can be produced either by paper folding and cutting or by drawing, most easily on isometric paper as here.



Example 11.5 **Footprint**

This is an old classic, which has variations:

We found some rather unusual footprints on the school field early this morning.

(The image would be enlarged to fill an A3 piece of paper.)

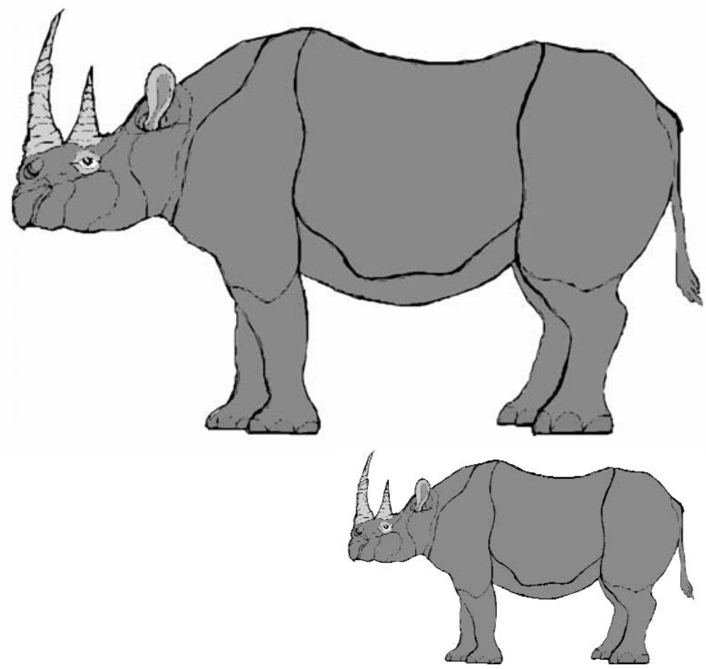
As you can see, they're rather large.

Someone said they saw 'a giant man' – that's all we know.

What conjectures can you make about the creature that produced them? Height, weight, walking / running speed, amount of food eaten in a day, depth of impression in damp soil, etc.

What evidence is there that this could be a hoax?

This can be a good opportunity to work on scaling up, with length/area/volume scale factors. The one artefact is enough to produce a lot of analysis.



'Crop circles' may provide another starting point for consideration of scale and perspective – as well as the topic of constructions, if you consider them to be of human origin!

Subsequent related lines of thought might include:

- If you climbed inside a 'doubling machine' that made all distances twice as big (not just perceptually but physically), how would you feel when you got out? Would walking feel the same or different? Why? Would you feel hotter or colder? Would you be stronger? Why?
- If the whole universe and everything in it 'doubled' (all distances) while you were asleep tonight, would you be able to tell in the morning? Why / why not?
- There are applications to biology, such as the effect on different animals of falling a certain height or the relative thickness of different animals' legs.
- There are obvious links with fiction in books such as *Gulliver's Travels* (Swift, 1726), *Alice's Adventures in Wonderland* (Carroll, 1865) and in films including *The Incredible Shrinking Man* (1957), *Fantastic Voyage* (1966) and *Honey, I Shrunk The Kids* (1989).